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F.L.R. effects on Rayleigh-Taylor instability of a plasma

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The effect of finite ion-Larmor radius is studied on the Rayleigh-Taylor instability of a plasma bounded by vacuum. The whole medium is embedded by a uniform magnetic field acting parallel to the gravity. The F.L.R. effects have been included through the stress tensor. The dispersion relation has been obtained and dissussed assuming that the inclusion of gyro-viscosity modifies the growth rates and introduces new instability modes which could be ignored. The growth rates have been obtained with the help of the computer and shown in figure 1. It is established that the gyro-viscosity reduces the instability of the system.

1. INTRODUCTION

It has been of considerable interest for quite some time to include the effect of finite ion-Larmor radius in the investigations of plasma instabilities. Many researchers have carried out the investigations with F.L.R. under different situations. Rosenbluth, Krall & Rostoker (1962) have studied the effect of F.L.R. on the gravitational instability using Vlasov - equations. Roberts & Taylor (1962) have obtained the same with the help of macroscopic equations. Singh & Hans (1966) have studied the F.L.R. effects on the interchange mode of Rayliegh-Taylor instability. Kalra (1967) has discussed the F.L.R. effects on the instability of superposed fluids. Nayyar & Trehan (1970) have investigated the effects of gyro-viscosity on the Rayleigh-Taylor instability of a plasma by including the F.L.R. effects through off-diagonal terms in the pressure tensor. The interchange mode has also been discussed.

All these authors have studied the problem for a horizontal magnetic field i.e., perpendicular to gravity. In this paper we have studied the F.L.R. effects on the Rayleigh-Taylor instability when the ambient magnetic field is parallel to gravity. It is supposed that the plasma is incompressible and is bounded by vacuum. It has been assumed that the gyro-viscosity is small i.e.

$$\frac{NTk^2}{nw_c} << 1,$$

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where N, T, k, n, and w_c denote the number density, the plasma temperature, the wave number, the frequency, the density and the ion gyro-frequency. It has been established that the instability of plasma vacuum boundary is reduced.

2. Basic Equations

We consider an infinitely conducting plasma of density ρ occupying half space $\infty > z > 0$. The space $-\infty < z < 0$ is vacuum. A uniform magnetic field is imposed in the direction of the z-axis vertically upwards. We shall assume that the medium is incompressible and inviscid. The hydromagnetic equations of motion are

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t}+(\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = -\nabla\pi+\frac{1}{4\pi}(\boldsymbol{H}.\nabla)\boldsymbol{H}+\boldsymbol{g}\rho-\nabla.p_m, \quad \ldots (1)$$

$$\boldsymbol{E}+\frac{1}{c}\boldsymbol{v}\times\boldsymbol{H}-\frac{1}{4\pi Ne}(\nabla\times\boldsymbol{H})\times\boldsymbol{H}+\frac{1}{Ne}\nabla p_e = 0, \quad \ldots (2)$$

$$\nabla.\boldsymbol{v} = 0, \qquad \nabla.\boldsymbol{H} = 0, \quad \ldots (3)$$

where

$$\pi = p+\frac{H^2}{8\pi}, \qquad \boldsymbol{g} = (0, 0, -g), \quad \ldots (4)$$

p_e is the electron pressure, p is the total fluid pressure, ρ is the constant density, $\boldsymbol{v}$ is the fluid velocity, $\boldsymbol{E}$ is electric field, $\boldsymbol{H}$ is the magnetic field. The displacement current is neglected in the Maxwell's equations. The anisotropic part of the pressure tensor p_m is due to the finite ion Larmor radius effects.

The relevant perturbed linearised equations of motion are

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\frac{1}{\rho}\Delta\pi_1+\frac{1}{4\pi\rho}(\boldsymbol{H}.\nabla)\boldsymbol{h}-\frac{1}{\rho}\nabla.\boldsymbol{P}, \quad \ldots (5)$$

$$\frac{\partial \boldsymbol{h}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{H})-c_1\nabla\times[(\nabla\times\boldsymbol{h})\times\boldsymbol{H}], \quad \ldots (6)$$

$$\nabla.\boldsymbol{h} = 0, \qquad \Delta.\boldsymbol{v} = 0, \quad \ldots (7)$$

where

$$\pi_1 = \frac{\delta p}{\rho}+\frac{\boldsymbol{H}.\boldsymbol{h}}{4\pi\rho},$$

$$c_1 = \frac{c}{4\pi Ne} = \frac{H}{4\pi\rho\omega_c}, \qquad \omega_c = \frac{eH}{mc}, \quad \ldots (8)$$

$$\boldsymbol{H} = (0, 0, \boldsymbol{H}),$$

$\boldsymbol{p}$, $\boldsymbol{h}$, $\boldsymbol{v}$ denote respectively the first order perturbations in the pressure tensor due to F.L.R., the magnetic field and velocity field, c is the speed of light and N is the number density.

The first order perturbation P in the stress tensor if the magnetic field is along the z-axis is given by

$$P_{xx} = -\rho\nu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad P_{yy} = \rho\nu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

$$P_{zz} = 0, \qquad P_{xy} = P_{yx} = \rho\nu\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right),$$

$$P_{xz} = P_{zx} = -2\rho\nu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right),$$

$$P_{yz} = P_{zy} = 2\rho\nu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right), \qquad \dots \quad (9)$$

$$\nu = \frac{NT}{4\omega_c}.$$

Taking curl of eqs. (5) and (6), we obtain

$$\frac{\partial}{\partial t}(\nabla\times\boldsymbol{v}) = \frac{1}{4\pi\rho}(\boldsymbol{H}\cdot\nabla)\nabla\times\boldsymbol{h} - \frac{1}{\rho}\nabla\times(\nabla\cdot\boldsymbol{P}), \qquad \dots \quad (10)$$

$$\frac{\partial}{\partial t}(\nabla\times\boldsymbol{h}) = (\boldsymbol{H}\cdot\nabla)\nabla\times\boldsymbol{v} - c_1(\boldsymbol{H}\cdot\nabla)\nabla\times\nabla\times\nabla\times\boldsymbol{h}. \qquad \dots \quad (11)$$

Some straight forward calculations give

$$\nabla\times(\nabla\cdot\boldsymbol{P}) = \rho\nu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\frac{\partial \boldsymbol{v}}{\partial z}. \qquad \dots \quad (12)$$

Eliminating $\boldsymbol{v}$ from eq. (11) with the help of eqs. (10), (12) and (6), we get

$$c_1H^2n - D^2\nabla\times\nabla\times\boldsymbol{h} - [V_A{}^2D^2 + \nu(k^2+2D^2)c_1HD - n^2]\times$$
$$\nabla\times\boldsymbol{h} - n\nu(k^2+2D^2)D\boldsymbol{h} = 0. \qquad \dots \quad (13)$$

where

$$D \equiv \frac{\mathrm{d}}{\mathrm{d}z}, \qquad V_A{}^2 = \frac{H^2}{4\pi\rho}.$$

Here the physical quantities have been assumed to vary as

$$f(z)\exp[ik_xx + ik_yy + nt]. \qquad \dots \quad (14)$$

At this stage we concern ourselves to the study of gyro-viscous effects only and so we drop the Hall-current term. The effect of Hall-current and resistivity has been studied by the authors (1973). Eq. (13) becomes

$$[V_A^2 D^2 - n^2]\nabla \times \boldsymbol{h} + n\nu(k^2 + 2D^2)D\boldsymbol{h} = 0 \qquad \ldots \quad (15)$$

3. The Method of Solutions

We assume a solution of the form

$$\boldsymbol{h} = \sum_{i=1}^{3} \boldsymbol{h}_i \qquad \ldots \quad (16)$$

where $\boldsymbol{h}_i$ satisfy

$$\Delta \times \boldsymbol{h}_i = \alpha_i \boldsymbol{h}_i, \qquad \ldots \quad (17)$$

and α_i are the roots of the equation

$$(n^2 - V_A^2 \lambda_i^2)\alpha_i = (k^2 + 2\lambda_i^2)\lambda_i, \qquad \ldots \quad (18)$$

and the expression for λ_i is obtained by solving eq. (17) as

$$\lambda_i^2 = k^2 - \alpha_i^2, \qquad k^2 = k_x^2 + k_y^2. \qquad \ldots \quad (19)$$

Writing the component of eq. (17) and making use of the divergence condition, we obtain

$$\boldsymbol{h}_i = C_i(p_i \mathbf{e}_x + q_i \mathbf{e}_y + \mathbf{e}_z)\exp(-\lambda_i z), \quad i = 1, 2, 3, \qquad \ldots \quad (20)$$

where

$$p_i = \frac{(ik_y \alpha_i - ik_x \lambda_i)}{k^2} \qquad \ldots \quad (21)$$

$$q_i = -(ik_y \lambda_i + ik_x \alpha_i)/k^2$$

The index i in eq. (20) does not imply summation.

The C_i are constants of integration which will be determined with the help of boundary conditions.

Having determined $\boldsymbol{h}$, we obtain $\boldsymbol{v}$ from eq. (6), neglecting the Hall term

$$\boldsymbol{v} = \Sigma \boldsymbol{v}_i = -\frac{1}{H\lambda_i}(n\boldsymbol{h}_i). \qquad \ldots \quad (22)$$

For the perturbation in pressure, we take z-component of eq. (5) and make use of eqs. (20) and (22) to obtain

$$p_1^i = \frac{\rho}{\lambda_i}\left[nv_z^i + \frac{H\lambda_i}{5\pi\rho} h_z^i + \frac{2\nu n}{H} h_z^i \alpha_i \right]$$

$$= \frac{\rho}{\lambda_i}\left[\frac{n^2}{H\lambda_i} + \frac{H\lambda_i}{4\pi\rho} + \frac{2\nu n}{H}\alpha_i \right] h_z^i, \quad i = 1, 2, 3, \qquad \ldots \quad (23)$$

the index i in the above equation does not imply summation.

The perturbation $\boldsymbol{h}^0$ in the magnetic field in the non-conducting fluid is governed by

$$curl\ \boldsymbol{h}^0 = 0, \qquad div\ \boldsymbol{h}^0 = 0, \qquad \ldots \quad (24)$$

whose solution is

$$\boldsymbol{h}^0 = C_4(\mathbf{e}_x ik_x + \mathbf{e}_y ik_y + k\mathbf{e}_z)e^{kz} \qquad 25)$$

where C_4 is a constant of integration to be determined with the help of boundary conditions.

4. The Boundary Conditions and the Dispersion Relation

The appropriate conditions to be satisfied at the perturbed boundary are

(*a*) At the interface $z = 0$, the normal component of the velocity must be compatible with the assumed form of the boundary

$$\delta z = \epsilon \exp(ik_x x + ik_y y + nt), \qquad (26)$$

where ϵ is a constant.

(*b*) The normal component of magnetic field must be continuous. The linearized form of this condition is

$$\Delta[\boldsymbol{h}.\boldsymbol{N}_0 + \boldsymbol{H}.\delta\boldsymbol{N}] = 0, \qquad (27)$$

where $\Delta[X]$ denotes the jump in X at the interface. Here $\boldsymbol{N}_0 = \boldsymbol{e}_z$ is the unit normal to the undisturbed interface and N denotes its displacement given by

$$\delta \mathrm{N} = -i\epsilon(\boldsymbol{e}_x k_x + \boldsymbol{e}_y k_y). \qquad \ldots \quad (28)$$

(*c*) Integrating the equation of motion (1) across the interface we obtain

$$N\Delta[\pi] = N.\Delta[\mathbf{BB} - \mathrm{P}]. \qquad \ldots \quad (29)$$

Multipying eq. (29) scalarly with $\delta\boldsymbol{N}$ we see that the normal stress must be continuous at the boundary, the linearized form of which is

$$\pi^p{}_1 - \epsilon g\rho + P_{zz} = \pi^v{}_1 = \frac{H^v k^v{}_z}{4\pi}, \qquad (30)$$

where p and v refer to plasma and vacuum and in wriring this use is made of

$$\frac{\partial \pi^p{}_0}{\partial z} = -g\rho.$$

The conditions (*d*) and (*e*) are obtained by taking the cross-product with N of eq. (29). This leads to the continuity of tangential stresses

$$(\boldsymbol{N}.\boldsymbol{B})\boldsymbol{N}\times(\nabla[\mathbf{B}]) = \boldsymbol{N}\times\nabla[\boldsymbol{N}.\boldsymbol{P}]. \qquad (31)$$

The linearized form of this condition gives

$$P_{zx} = 0$$

and

$$\Delta[H(h_z + H\delta N_z)] = P_{yz}. \quad \ldots (32)$$

Making use of the above five boundary conditions we obtain following five equations containing C_i's and ϵ

$$-\sum_{i=1}^{3} \frac{C_i}{\lambda_i} = \epsilon H, \quad \ldots (33)$$

$$\sum_{i=1}^{v} C_i = kC_4, \quad \ldots (34)$$

$$\sum_{i=1}^{3} C_i q_i - \sum_{i=1}^{3} ik_y \frac{C_i}{\lambda_i} = 0, \quad \ldots (35)$$

$$H\Sigma C_i - H_v kC_4 = 2\nu\rho\left(\Sigma C_i p_i - ik_x \Sigma \frac{C_i}{\lambda_i}\right) \frac{n}{H} = 0, \quad \ldots (36)$$

$$\sum_{i=1}^{3}\left(\frac{n^2}{\lambda_i^2} + V_A^2 + 2\nu \frac{n\alpha_i}{\lambda}\right) C_i - \epsilon g H = \frac{HH_v}{4\pi\rho} kC_4. \quad \ldots (37)$$

The dispersion relation is obtained by setting the determinant of the coefficients of C_i and ϵ to zero. As the magnetic field is vertical there cannot be any discontinuity in it at the interface. Eqs. (33—36) yield the dispersion relation

$$\begin{aligned}
&L_1^2\{(\lambda_1^2+k^2)(\alpha_2-\alpha_3)-\alpha_1\lambda_1(\lambda_2-\lambda_3)(1+k^2/\lambda_2\lambda_3)\} \\
&\quad +\lambda_1(L_1-V_A^2)\{(\alpha_2-\alpha_3)(\lambda_1^2+k^2)/\lambda_1+(\alpha_3-\alpha_1)(\lambda_2^2+k^2)/\lambda_2 \\
&\quad +(\alpha_1-\alpha_2)(\lambda_3^2+k^2)/\lambda_3\}+L_2\{(\lambda_1^2+k^2)(\alpha_3-\alpha_1) \\
&\quad -\alpha_1\lambda_1(\lambda_3-\lambda_1)(1+k^2/\lambda_1\lambda_3)\}+L_3\{(\lambda_1^2+k^2)(\alpha_1-\alpha_2) \\
&\quad -\alpha_1\lambda_1(\lambda_1-\lambda_2)(1+k^2/\lambda_1\lambda_3)\}+g[(\lambda_1-\lambda_2)(\alpha_1-\alpha_3) \\
&\quad -(\lambda_1-\lambda_3)(\alpha_1-\alpha_2)-\frac{1}{\lambda_1\lambda_2\lambda_3}\{\lambda_3(\lambda_1-\lambda_2)(\alpha_3\lambda_1-\alpha_1\lambda_3) \\
&\quad -\lambda_2(\lambda_1-\lambda_3)(\alpha_2\lambda_1-\alpha_1\lambda_2)\}] = 0, \quad \ldots (38)
\end{aligned}$$

where

$$L_j = n^2/\lambda_j^2 + V_A^2 + 2\nu\alpha_j n/\lambda_j, \quad j = 1, 2, 3, \quad \ldots (39)$$

$$V_A^2 = H_p^2/4\pi\rho.$$

The equation giving $t = \alpha^2/k^2$ is

$$t^3(\beta^4+4\eta^2)-t^2[2\beta^2(\beta^2-1)+16\eta^2]+t[(\beta^2-1)^2+21\eta^2]-g\eta^2 = 0, \quad \ldots \quad (40)$$

where

$$\beta = \frac{kV_A}{n}, \qquad \eta = \frac{\nu k^2}{n}.$$

The three roots of this equation are obtained by iteration and are approximately given by

$$t_1 = \eta^2 A^2, \qquad t_{2,3} = a\left[1+\eta^2\frac{b}{a} \pm \eta\frac{c}{2a}\right],$$

$$\lambda_1 = 1-\eta^2 A^2/2, \qquad \lambda^2_{2,3} = \beta^{-2}[1-\eta^2 b\beta^2 \mp \eta c\beta^2], \quad \ldots \quad (41)$$

where

$$A = 3/(\beta^2-1), \qquad a = (\beta^2-1)/\beta^2,$$

$$b = \frac{(\beta^2-1)}{\beta^2}\left[-\frac{4}{\beta^4}+\frac{8}{\beta^2(\beta^2-1)}+\frac{9\beta^2}{2(\beta^2-1)^3}\right],$$

$$c = (\beta^2+2)/\beta^4(\beta^2-1)^{\frac{1}{2}}.$$

Making use of eqs. (39), (41) and (42), dispersion relation (38) reduces to

$$\frac{gk}{n^2}\left[1-\frac{1}{\beta^2}+\eta^2\left\{b\beta+\beta(\beta^2+1)\left(\frac{b}{2}+\frac{c^2\beta^2}{8}\right)-A^2\beta+c\beta^2Aa^{-\frac{1}{2}}\right.\right.$$

$$\left.\left.-\frac{c^2\beta}{4}\left(\frac{1+\beta^2}{a}-\beta^4\right)\right\}\right] +4-\beta-\frac{1}{\beta}-2\beta^2$$

$$\eta^2\left[2\left(A^2+\frac{2b}{a}\right)-\beta\left\{\left(1+\frac{1}{\beta^2}\right)\left(-\frac{b}{a}+\frac{A^2}{2}+\right.\right.\right.$$

$$\left.+\frac{b\beta^2}{2}+\frac{c^2\beta^2}{4}\right)-\frac{b}{2}-\frac{c^2\beta^2}{8}\Big\}+\beta^2\Big\{A^2-\frac{2b}{a}$$

$$+\beta^2\left(b+\frac{c^2\beta^2}{4}\right)\Big\}-\left(\beta+\frac{1}{\beta}\right)\left(2A+\frac{c^2\beta^4}{4}\right)$$

$$+\frac{Ac^2}{4}\beta^4+8A-\frac{c\beta}{2}\left(\frac{c}{2a}-Aa^{-\frac{1}{2}}\right)-\frac{c\beta}{2}\left(\frac{A}{a}+\frac{c\beta^2}{2a^{\frac{1}{2}}}\right)$$

$$-\frac{Ac\beta^3a^{-\frac{1}{2}}}{2}(1+\beta^2)+\frac{3}{2}c^2\beta^4+2ca^{\frac{1}{2}}\beta\left(\frac{1}{a}+\beta^2\right)$$

$$\left.+2\beta(A(3+\beta)-Ca^{-\frac{1}{2}})+c\beta^4(A(3+\beta)-Ca^{-\frac{1}{2}})a^{-\frac{1}{2}}\right] = 0. \quad \ldots \quad (43)$$

In the absence of gyro-viscosity the dispersion relation is

$$D_0 = (1-\omega^2)\frac{gk}{\Omega^2_A} - 2\omega - \omega^2 + 4\omega^3 - \omega^4 = 0 \qquad \ldots (44)$$

where

$$\omega = 1/\beta, \qquad \Omega_A^2 = k^2 V_A^2. \qquad \ldots (45)$$

In the dispersion relation (43) we retain the powers of $1/\beta$ up to four only assuming that the inclusion of gyro-viscosity changes the growth rates and introduces new instability modes which could be ignored. In the absence of gyro-viscosity the D.R. is of the fourth degree in ω.

The D.R. (43) then becomes

$$D_0 + \frac{\omega_\nu^2}{\Omega_A^2}\left[\frac{217}{4}\frac{gk}{\Omega_A^2} + \frac{209}{4}\omega + \frac{343}{4}\omega^2 - \frac{127}{4}\omega^3 + \frac{617}{4}\omega^4\right] = 0. \qquad \ldots (46)$$

Table showing all the roots of ω^2

k^*	$\eta^* = 0$		$\eta^* = 0.1$		$\eta^* = 0.01$	
	$\omega^*_R{}^2$	$\omega_0{}^2$	$\omega^*_R{}^2$	$\omega_I{}^2$	$\omega^*_R{}^2$	$\omega_I{}^2$
0.1	0.0489	0	−0.2168	0.1871	0.0611	0
	−0.5791	0	−0.2168	−0.1871	−0.8069	0
	3.5301	0	0.1554	1.0687	15.989	0
	1.0000	0	0.1554	−1.0687	0.8550	0
0.5	0.2331	0	−0.3660	0.5713	0.3291	0
	−0.6312	0	−0.3660	−0.5713	−0.8597	0
	3.3981	0	0.3045	0.9729	15.878	0
	1.0000	0	0.3045	−0.9729	0.7503	0
1.0	0.4608	0	−0.5339	0.7289	15.738	0
	0.6751	0	−0.5339	−0.7289	−0.9057	0
	3.2143	0	0.4725	0.9742	0.6329	0.2691
	0.9999	0	0.4725	−0.9742	0.6329	−0.2691
1.5	0.7071	0	−0.6452	0.8089	15.595	0
	−0.7071	0	−0.6452	−0.8089	−0.9396	0
	3.0000	0	0.5837	1.0001	0.7213	0.4120
	1.0000	0	0.5837	−1.0001	0.7213	−0.4120
2.0	1.0000	0	−0.7288	0.8336	15.449	0
	−0.7320	0	−0.7288	−0.8636	−0.9662	0
	2.7320	0	0.6674	1.0264	0.8075	0.5012
	0.9999	0	0.6674	−1.0264	0.8078	−0.5012
2.5	0.9999	0	−0.7967	0.9056	15.3000	0
	−0.7523	0	−0.7967	−0.9056	−0.9879	0
	2.3200	0	0.7353	1.0505	0.8926	0.5641
	1.4323	0	0.7353	−1.0505	0.8926	−0.5641
3.0	0.9999	0	−0.8545	0.9401	15.149	0
	−0.7692	0	−0.8545	−0.9401	−1.0068	0
	1.8846	0.5897	0.7930	1.0724	0.9775	0.6096
	1.8846	−0.5897	0.7930	−1.9775	0.9775	−0.6096

Dispersion relation (46) has been solved numerically for the following values of $k^* = gk/\Omega_A^2$ and ω_ν^2/Ω_A^2 on the computer

$$k^* = \frac{gk^*}{\Omega_A^2} = 0{\cdot}1,\ 0{\cdot}5,\ 1,\ 1{\cdot}5,\ 2,\ 2{\cdot}5,\ 3{\cdot}0$$

$$\eta^* = \frac{\omega_\nu^2}{\Omega_A^2} = 0,\ 0{\cdot}1,\ 0{\cdot}02.$$

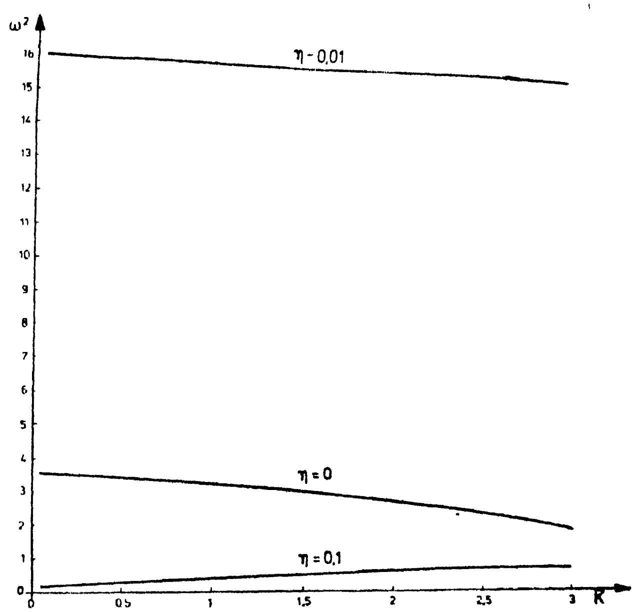

Fig. 1 W. K. relation showing the largest positive ω

The roots of eq. (46) with largest positive real part for various cases have been shown in figure 1. Curves 1, 2, 3 correspond to $\eta^* = 0$, $0{\cdot}01$ and $0{\cdot}1$ respectively. It is inferred from the curves that for small ($\eta^* = 0{\cdot}01$) the growth rate is larger than in the case $\eta^* = 0$ while for $\eta^* = 0{\cdot}1$, the growth rate is smaller than that for $\eta^* = 0$. This implies that the effect of large gyro-viscosity is to decrease the growth rate.

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